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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
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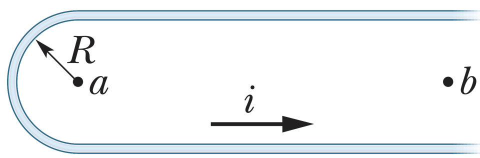
Problem Set 6

Magnetic Fields due to Currents

Problem 1. *In the Figure below, a current $i = 10\,\mathrm{A}$ is set up in a long hairpin conductor formed by bending a wire into a semicircle of radius $R = 5.0\,\mathrm{mm}$. Point b is midway between the straight sections and so distant from the semicircle that each straight section can be approximated as being an infinite wire. What are the **(a)** magnitude and **(b)** direction (into or out of the page) of $\vec{B}$ at a and the **(c)** magnitude and **(d)** direction of $\vec{B}$ at b?*



Solution. **(a)** We find the field by superposing the results of two semi-infinite wires and a semicircular arc. The direction of $\vec{B}$ is out of the page according to the right-hand rule. The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i \pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right) = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{2(0.0050\,\mathrm{m})}\left(\frac{1}{\pi} + \frac{1}{2}\right) = 1.0 \times 10^{-3}\,\mathrm{T}$$

upon substituting $i = 10\,\mathrm{A}$ and $R = 0.0050\,\mathrm{m}$.

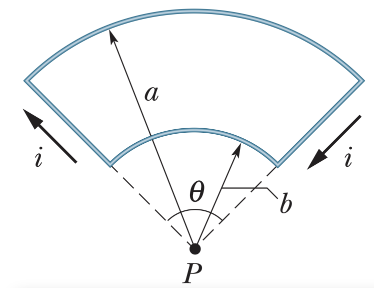
(b) The direction of this field is out of the page.

(c) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{\pi(0.0050\,\mathrm{m})} = 8.0 \times 10^{-4}\,\mathrm{T}$$

(d) This field, too, points out of the page. □

Problem 2. *In the Figure below, two circular arcs have radii* $a = 13.5\,\mathrm{cm}$ *and* $b = 10.7\,\mathrm{cm}$*, subtend angle* $\theta = 74.0\,°$*, carry current* $i = 0.411\,\mathrm{A}$*, and share the same center of curvature* P*. What are the* ***(a)*** *magnitude and* ***(b)*** *direction (into or out of the page) of the net magnetic field at* P*?*

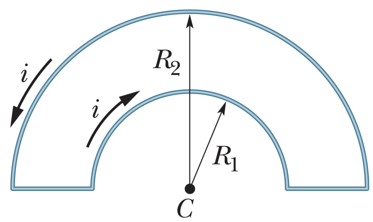


Solution. **(a)** We see that the current in the straight segments collinear with P do not contribute to the field at that point. Using the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$\vec{B} = \frac{\mu_0 i\theta}{4}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.411\,\mathrm{A})(74\,° \cdot \pi/180\,°)}{4\pi}\left(\frac{1}{0.107\,\mathrm{m}} - \frac{1}{0.135\,\mathrm{m}}\right)$$
$$= 1.02 \times 10^{-7}\,\mathrm{T}$$

(b) The direction is out of the page. □

Problem 3. *In the Figure below, two semicircular arcs have radii* $R_2 = 7.80\,\mathrm{cm}$ *and* $R_1 = 3.15\,\mathrm{cm}$*, carry current* $i = 0.281\,\mathrm{A}$*, and have the same center of curvature* C*. What are the* ***(a)*** *magnitude and* ***(b)*** *direction (into or out of the page) of the net magnetic field at* C*?*

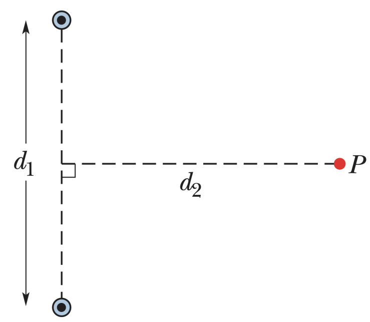


Solution. **(a)** The net field at C is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(0.281\,\mathrm{A})}{4}\left(\frac{1}{0.0315\,\mathrm{m}} - \frac{1}{0.0780\,\mathrm{m}}\right) = 1.67 \times 10^{-6}\,\mathrm{T}$$

(b) The direction of the field is into the page. □

Problem 4. *The Figure shows two very long straight wires (in cross section) that each carry a current of* $4.00\,\mathrm{A}$ *directly out of the page. Distance* $d_1 = 6.00\,\mathrm{m}$ *and distance* $d_2 = 4.00\,\mathrm{m}$. *What is the magnitude of the net magnetic field at point* P, *which lies on a perpendicular bisector to the wires?*



Solution. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the y-axis (passing through P), consisting of two equal magnetic field y-components.

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where $i = 4.00\,\mathrm{A}$, $r = \sqrt{d_2^2 + d_1^2/4} = 5.00\,\mathrm{m}$, and

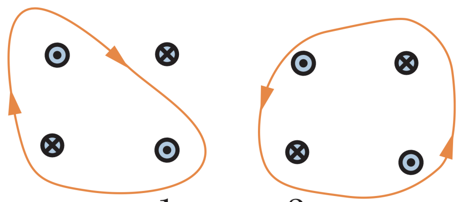
$$\theta = \arctan\left(\frac{d_2}{d_1/2}\right) = \arctan\left(\frac{4.00\,\mathrm{m}}{6.00\,\mathrm{m}/2}\right) = \arctan\left(\frac{4}{3}\right) = 53.1\,^\circ$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(4.00\,\mathrm{A})}{\pi(5.00\,\mathrm{m})}\sin 53.1\,^\circ = 2.56 \times 10^{-7}\,\mathrm{T}$$

□

Problem 5. *Each of the eight conductors in the Figure below carries* 2.0 A *of current into or out of the page. Two paths are indicated for the line integral* $\oint \vec{B} \cdot d\vec{s}$*. What is the value of the integral for* ***(a)*** *path 1 and* ***(b)*** *path 2?*



Solution. The value of the line integral $\oint \vec{B} \cdot d\vec{s}$ is proportional to the net current enclosed. By Ampere's law, we have $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where i_{enc} is the current enclosed by the closed path.

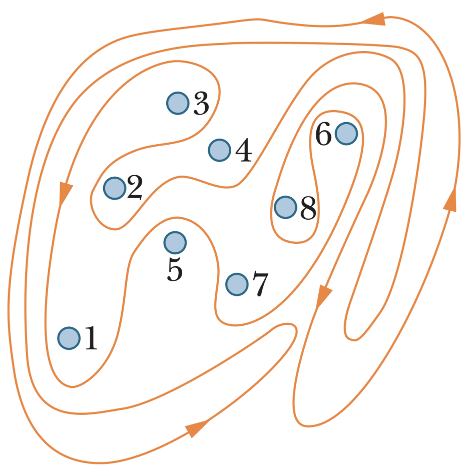
(a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "Amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2.0\,\text{A}) = -2.5 \times 10^{-6}\,\text{T} \cdot \text{m}$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), thus $\oint \vec{B} \cdot d\vec{s} = -\mu_0 i_{\text{enc}} = 0$.

DISCUSSION The value of $\oint \vec{B} \cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the Amperian loop. $\square$

Problem 6. *Eight wires cut the page perpendicularly at the points shown in the Figure. A wire labelled with the integer* k $(k = 1, 2, ..., 8)$ *carries the current* $k \cdot i$*, where* $i = 4.50\,\mathrm{mA}$*. For those wires with odd* k*, the current is out of the page; for those with even* k*, it is into the page. Evaluate* $\oint \vec{B} \cdot d\vec{s}$ *along the closed path indicated and in the direction shown.*



Solution. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0(7i - 6i + 3i + i) = 5\mu_0 i = 5(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(4.50 \times 10^{-3}\,\mathrm{A}) = 2.83 \times 10^{-8}\,\mathrm{T \cdot m}$$

□

Problem 7. *The current density $\vec{J}$ inside a long, solid, cylindrical wire of radius $a = 3.1\,\text{mm}$ is in the direction of the central axis, and its magnitude varies linearly with radial distance r from the axis according to $J = J_0 r/a$, where $J_0 = 310\,\text{A/m}^2$. Find the magnitude of the magnetic field at* ***(a)*** *$r = 0$,* ***(b)*** *$r = a/2$, and* ***(c)*** *$r = a$.*

Solution. For $r \le a$,

$$B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J(r) 2\pi r dr = \frac{\mu_0}{2\pi r}\int_0^r J_0 \frac{r}{a} 2\pi r dr = \frac{\mu_0 J_0 r^2}{3a}$$

(a) At $r = 0$, $B = 0$.

(b) At $r = a/2$, we have

$$B(r) = \frac{\mu_0 J_0 r^2}{3a} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times 10^{-3}\,\text{m}/2)^2}{3(3.1\times 10^{-3}\,\text{m})} = 1.0\times 10^{-7}\,\text{T}$$

(c) At $r = a$,

$$B(r = a) = \frac{\mu_0 J_0 a}{3} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times 10^{-3}\,\text{m})}{3} = 4.0\times 10^{-7}\,\text{T}$$

□

Problem 8. *A solenoid that is* $95.0\,\mathrm{cm}$ *long has a radius of* $2.00\,\mathrm{cm}$ *and a winding of 1200 turns; it carries a current of* $3.60\,\mathrm{A}$*. Calculate the magnitude of the magnetic field inside the solenoid.*

Solution. For the ideal solenoid, which does not make use of the coil radius, is the preferred method:

$$B = \mu_0 i n = \mu_0 i \left(\frac{N}{l}\right)$$

where $i = 3.60\,\mathrm{A}$, $l = 0.950\,\mathrm{m}$, and $N = 1200$. This yields $B = 0.00571\,\mathrm{T}$. $\square$

Problem 9. *A long solenoid has* $100\,\text{turns/cm}$ *and carries current* i*. An electron moves within the solenoid in a circle of radius* $2.30\,\text{cm}$ *perpendicular to the solenoid axis. The speed of the electron is* $0.0460c$ *(*c *is the speed of light). Find the current* i *in the solenoid. (Hint mass of electron is* $9.11 \times 10^{-31}\,\text{kg}$*, speed of light is* $3.00 \times 10^{8}\,\text{m/s}$*.)*

Solution. The orbital radius for the electron is

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$\begin{aligned} i &= \frac{mv}{e\mu_0 nr} = \frac{(9.11 \times 10^{-31}\,\text{kg})(0.0460)(3.00 \times 10^{8}\,\text{m/s})}{(1.60 \times 10^{-19}\,\text{C})(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(100/0.01\,\text{m})(2.30 \times 10^{-2}\,\text{m})} \\ &= 0.272\,\text{A} \end{aligned}$$

□

Problem 10. *A long solenoid with* $10.0\,\mathrm{turns/cm}$ *and a radius of* $7.00\,\mathrm{cm}$ *carries a current of* $20.0\,\mathrm{mA}$*. A current of* $6.00\,\mathrm{A}$ *exists in a straight conductor located along the central axis of the solenoid.* ***(a)*** *At what radial distance from the axis will the direction of the resulting magnetic field be at* $45.0\,^\circ$ *to the axial direction?* ***(b)*** *What is the magnitude of the magnetic field there?*

Solution. The net field at a point inside the solenoid is the vector sum of the fields of the solenoid and that of the long straight wire along the central axis of the solenoid. The magnetic field at a point P is given by $\vec{B} = \vec{B}_s + \vec{B}_w$, where $\vec{B}_s$ and $\vec{B}_w$ are the fields due to the solenoid and the wire, respectively. The direction of $\vec{B}_s$ is along the axis of the solenoid, and the direction of $\vec{B}_w$ is perpendicular to it, so the two fields are perpendicular to each other, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at $45\,^\circ$ with the axis, we must have $\vec{B}_s = \vec{B}_w$.

(a) Thus,

$$B_s = B_w \quad \Rightarrow \quad \mu_0 i_s n = \frac{\mu_0 i_w}{2\pi d}$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\mathrm{A}}{2\pi(20.0\times 10^{-3}\,\mathrm{A})(10\,\mathrm{turns/cm})} = 4.77\,\mathrm{cm}$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}(4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(20.0\times 10^{-3}\,\mathrm{A})(10\,\mathrm{turn}/0.01\,\mathrm{m}) = 3.55\times 10^{-5}\,\mathrm{T}$$

(DISCUSSION) In general, the angle $\vec{B}$ makes with the solenoid axis is give by

$$\phi = \arctan\left(\frac{B_w}{B_s}\right) = \arctan\left(\frac{\mu_0 i_w/2\pi d}{\mu_0 i_s n}\right) = \arctan\left(\frac{i_w}{2\pi d n i_s}\right)$$

□